Cart3D Shape Optimization and Lift Distributions

Alex Gary – Cal Poly

VSP Workshop
August 9, 2013
Overview

- Application: Sears-Haack Fixed Diameter Body
- Lift Superposition and Aerodynamic Influence Coefficient (AIC) Matrices
- Generating an AIC with Cart3D
- Application: Matching Lift Distributions on Subsonic Wing
- Future Work
Objective: Minimize CD

VSP Model:
- Fuselage component with one circular cross section

Variables:
- Top tangent angle and both tangent strengths at the circular cross section
- Streamwise location of the circular cross section
- Top tangent angle and one tangent strength at each endpoint
- All other tangent angles and strengths were linked to make it a symmetric body of revolution
- Total of 8 design variables
Results

Initial Body

Optimized Body
Comparison to Theoretical Result

Fixed Diameter Sears-Haack Body

Optimized Body

<table>
<thead>
<tr>
<th></th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>.889</td>
</tr>
<tr>
<td>Final</td>
<td>.271</td>
</tr>
<tr>
<td>Fixed Diameter</td>
<td>.282</td>
</tr>
</tbody>
</table>
Lift Superposition

Using lift superposition to match a desired lift distribution

• Choose a set of basis functions for the twist distribution
• Find the change in the lift distribution with respect to each of the basis twist functions
• Construct an aerodynamic influence coefficient matrix (AIC)

\[
AIC = \begin{bmatrix}
\frac{\partial l_1}{\partial \phi_1} & \frac{\partial l_1}{\partial \phi_2} & \cdots & \frac{\partial l_1}{\partial \phi_N} \\
\frac{\partial l_2}{\partial \phi_1} & \frac{\partial l_2}{\partial \phi_2} & \cdots & \frac{\partial l_2}{\partial \phi_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial l_M}{\partial \phi_1} & \frac{\partial l_M}{\partial \phi_2} & \cdots & \frac{\partial l_M}{\partial \phi_N}
\end{bmatrix}
\]

• Find the weights of the twist basis functions for a target lift distribution

\[
\vec{w} = AIC^{-1}\vec{l}_t
\]
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\end{bmatrix}
\]

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\vec{w} = AIC^{-1}\vec{l_t}
\]
Triload Method

VSP

Cart3D

Perl Script

Triload
Discretized Loads Method

\[ \begin{bmatrix} \frac{\partial l_1}{\partial \phi_1} \\ \frac{\partial l_2}{\partial \phi_1} \\ \vdots \\ \frac{\partial l_M}{\partial \phi_1} \end{bmatrix} \]

Perl Script

# Cart3D CLIC2 output for Untitled configuration
# Fri Jul 5 13:36:26 2013
# section_1 Axial Force (C_A): -0.0047950673
section_1 Lateral Force (C_Y): 1.5088040e-19
section_1 Normal Force (C_N): -0.0016678738
# section_1 Drag Force (C_D): 0.00797950673
section_1 Side Force (C_S): -3.5088040e-19
section_1 Lift Force (C_L): -0.006166787385
section_10 Axial Force (C_A): -0.000099595721
section_10 Lateral Force (C_Y): 0.0027517773
section_10 Normal Force (C_N): -0.0005851528
# section_10 Drag Force (C_D): 0.00099595721
section_10 Side Force (C_S): -0.0027517773
section_10 Lift Force (C_L): -0.0005851528

VSP

Cart3D

Clic
Computational Cost

Using Adaptations:

- CFD solution on final mesh = 1
- Adjoint on final mesh = 1
- CFD solutions during adaption process = 1
- Adjoint solutions during adaption process = 1
- Total cost for adapted solution without adjoint = 3

<table>
<thead>
<tr>
<th>AIC Method</th>
<th>Equivalent Final Mesh CFD Solves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triload</td>
<td>3*(Number of Shape Functions+1)</td>
</tr>
<tr>
<td>Adjoint</td>
<td>3+Number of Lift Sections</td>
</tr>
<tr>
<td>Discretized Loads</td>
<td>3*(Number of Shape Functions+1)</td>
</tr>
</tbody>
</table>
Subsonic Lift Distribution Matching

Constant Properties:
- Mach: 0.3
- AR: 7
- Airfoil: NACA 0012

Variables:
- Twist at 5 evenly spaced stations along the span
- Alpha
- Total of 6 Basis Shape Functions
Comparison of AIC Matrices
Elliptical Distribution Results

Target Distribution: Elliptical
Target CL: 0.5
Results:

<table>
<thead>
<tr>
<th></th>
<th>Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sect 1</td>
<td>0.252</td>
</tr>
<tr>
<td>Sect 2</td>
<td>0.266</td>
</tr>
<tr>
<td>Sect 3</td>
<td>1.255</td>
</tr>
<tr>
<td>Sect 4</td>
<td>1.020</td>
</tr>
<tr>
<td>Sect 5</td>
<td>6.263</td>
</tr>
<tr>
<td>Alpha</td>
<td>6.959</td>
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Elliptical Distribution Results

Target Distribution: Elliptical
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<tr>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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Triangular Distribution Results

Target Distribution: Triangular
Target CL: 0.2

Results:

<table>
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<tr>
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<tbody>
<tr>
<td>Sect 1</td>
<td>3.014</td>
</tr>
<tr>
<td>Sect 2</td>
<td>3.656</td>
</tr>
<tr>
<td>Sect 3</td>
<td>4.796</td>
</tr>
<tr>
<td>Sect 4</td>
<td>5.738</td>
</tr>
<tr>
<td>Sect 5</td>
<td>7.576</td>
</tr>
<tr>
<td>Alpha</td>
<td>6.431</td>
</tr>
</tbody>
</table>
**Triangular Distribution Results**

Target Distribution: Triangular
Target CL: 0.2

### Results:

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<tr>
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<td>6.431</td>
</tr>
</tbody>
</table>

![Graph showing twist distribution](image-url)
Limitations

- Cl vs α curve must be linear
- Transonic Example:
  - Mach: .7
  - Target CL: 0.5
  - Target Distribution: Elliptical
- Results:

<table>
<thead>
<tr>
<th>Sect</th>
<th>Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sect 1</td>
<td>0.175</td>
</tr>
<tr>
<td>Sect 2</td>
<td>-0.018</td>
</tr>
<tr>
<td>Sect 3</td>
<td>0.946</td>
</tr>
<tr>
<td>Sect 4</td>
<td>-0.093</td>
</tr>
<tr>
<td>Sect 5</td>
<td>4.477</td>
</tr>
<tr>
<td>Alpha</td>
<td>5.134</td>
</tr>
</tbody>
</table>
Cl vs $\alpha$ NACA 0012

Compound Objective Problem Statement and Setup

Goal: Use compound objective to match elliptical lift distribution.

Objective: \[ \text{minimize} \sum_{i=1}^{10} (CL_i - T_i)^2 \]

Variables:
- Twist at 5 evenly spaced stations along the span
- Alpha

Constant Properties:
- Mach: 0.3
- AR: 7
- Airfoil: NACA 0012
### Results of Optimization

<table>
<thead>
<tr>
<th></th>
<th>Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sect 1</td>
<td>-1.461</td>
</tr>
<tr>
<td>Sect 2</td>
<td>-1.332</td>
</tr>
<tr>
<td>Sect 3</td>
<td>-1.082</td>
</tr>
<tr>
<td>Sect 4</td>
<td>-7.029</td>
</tr>
<tr>
<td>Sect 5</td>
<td>-2.181</td>
</tr>
<tr>
<td>Alpha</td>
<td>5.063</td>
</tr>
</tbody>
</table>
Comparison to AIC Solution
Second Optimization

Goal: Match an elliptical distribution

Objective: \( \text{minimize } C_D + (C_L - 0.5)^2 \)

Variables:
- Twist at 5 evenly spaced stations along the span
- Alpha

Constant Properties:
- Mach: 0.3
- AR: 7
- Airfoil: NACA 0012
## Results of Optimization

<table>
<thead>
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<tr>
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<td>-1.225</td>
</tr>
<tr>
<td>Sect 2</td>
<td>-1.084</td>
</tr>
<tr>
<td>Sect 3</td>
<td>-1.002</td>
</tr>
<tr>
<td>Sect 4</td>
<td>-0.815</td>
</tr>
<tr>
<td>Sect 5</td>
<td>-0.323</td>
</tr>
</tbody>
</table>

Alpha = 4.685

![Graph showing twist values for different sections]
Comparison to AIC Solution

[Graph showing Comparison to AIC Solution and Drag Minimization Solution]
Computational Cost

Optimization Cost:
  • Using adaptation each design iteration costs 4

<table>
<thead>
<tr>
<th>Matching Method</th>
<th>Equivalent Final Mesh CFD Solves</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC Triload</td>
<td>3*(6 Shape Functions +1) = 21</td>
</tr>
<tr>
<td>Compound Objective</td>
<td>4*(4 design iterations) = 16</td>
</tr>
<tr>
<td>Twist to Minimize Drag</td>
<td>4*(11 design iterations) = 44</td>
</tr>
</tbody>
</table>
Future Work

Motivation:
- Propulsion-Airframe Integration (PAI) using a linear equality constraint on lift distribution
- Linear equality constraints reduce problem dimensionality
- Linear equality constraints are satisfied at every iteration

Constraint Formulation:
- \[ AIC\vec{w} = \vec{b} \]
- \[ \vec{b} = \vec{l}_d - \vec{l}_n - \vec{l}_w \]

Progress:
- Modified Cart3D-SNOPT interface to allow linear equality constraints
- Ran Cart3D QP example problem with linear constraints
- Linear constraints reduced number of iterations from 7 to 4
Future Work

Current Problem:

\[ AIC \vec{w} = \vec{b} \]
is not a true linear equality constraint because

\[ \vec{b} = \vec{l}_d - \vec{l}_n - \vec{l}_w \]
is nonlinear

Proposed Solution:

- Iteratively run the optimizer
Questions?